A Proof System for the Linear Time μ-Calculus

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Speaker: Christian Dax

Motivation: Why Linear-Time μ-Calculus?

Context

- μTL is a temporal logic like *LTL*
- used for specification of properties of systems (safety, fairness)
- need for efficient algorithms for model-checking and validity-checking
  - μTL formula not valid: counter-example
  - μTL formula is valid: proof object as "certificate"

Weakness of LTL

- *LTL* strictly less expressive than μTL
- μTL can express ω-regular properties
- no counting in *LTL*: "every *n*th step *p* should hold"

Outline

Linear Time μ-Calculus

Proof System

Automatic Proof-Search

Experimental Results

Conclusion

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Syntax and Semantics

Syntax:

$$\varphi ::= \underbrace{p \mid \neg p \mid \varphi \wedge \varphi \mid \varphi \vee \varphi}_{\text{propositional logic}} \mid \underbrace{\bigcirc\varphi}_{\text{next}} \mid X \mid \underbrace{\mu X.\varphi}_{\text{least fixp.}} \mid \underbrace{\nu X.\varphi}_{\text{greatest fixp.}}$$

$p \in \mathcal{P}$ set of propositions
$X \in \mathit{Vars}$ set of variables

Semantics:

- interpreted over infinite $2^{\mathcal{P}}$-words, e.g. $\{p, \neg q\}\{p, \neg q\}\{q\}^\omega$
- propositional part and $\bigcirc$-operator as in LTL

Example

$$\{p\}\{p\}\{q\}^\omega \quad \models \quad p \wedge \bigcirc((p \vee q) \wedge \bigcirc q)$$

Semantics of Fixpoints

Least Fixpoint: “finite repetition”

- $\mu X.\varphi \quad \approx \quad \bigvee_{k \in \mathbb{N}} \mu^k X.\varphi$

$$\mu^0 X.\varphi := \mathit{false}$$
$$\mu^{i+1} X.\varphi := \varphi[\mu^i X.\varphi / X]$$

Example

$$\begin{aligned} \{\neg p\}^k \{p\} \{\neg p\}^\omega \quad & \models \quad \mu X.p \vee \bigcirc X \\ & \approx \quad p \vee \bigcirc p \vee \bigcirc \bigcirc p \vee \cdots \vee (\underbrace{\bigcirc \bigcirc \ldots \bigcirc}_{k \text{ times}} p) \end{aligned}$$

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Semantics of Fixpoints

Greatest Fixpoint: "infinite repetition"

- $\nu X.\varphi \quad \approx \quad \bigwedge_{k\in\mathbb{N}} \nu^k X.\varphi$

$$\nu^0 X.\varphi := \mathit{true}$$
$$\nu^{i+1} X.\varphi := \varphi[\nu^i X.\varphi / X]$$

Example

- $$\begin{aligned} \{p\}^\omega \quad &\models \quad \nu X.p \wedge \bigcirc X \\ &\approx \quad p \wedge \bigcirc p \wedge \bigcirc \bigcirc p \wedge \dots \end{aligned}$$
- $\nu X.p \wedge \bigcirc \bigcirc X$: "p at odd positions"

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Previous Work

Our proof system is similar to ...

- tableau systems used by Stirling, Kaivola, Bradfield, Esparza, Mader: rather theoretical than practical because Savitch's theorem "NSPACE($f(n)$) $\subseteq$ DSPACE($f^2(n)$)" used.
- Street/Emerson's work, adapted to μTL by Vardi: similar idea, but more complicated representation. No explicit construction, no implementation.

Our aim: practical decision procedure.

Question: Is φ Valid?

Construction of Gentzen-style Proof:

- start at bottom with $\vdash \varphi$
- Step 1: build infinite tree by rules (bottom-up)

$$\frac{\vdash \varphi, \psi, \Gamma}{\vdash \varphi \vee \psi, \Gamma} \qquad \frac{\vdash \varphi, \Gamma \qquad \vdash \psi, \Gamma}{\vdash \varphi \wedge \psi, \Gamma}$$

$$\frac{\vdash \varphi[\sigma X.\varphi / X], \Gamma}{\vdash \sigma X.\varphi, \Gamma} \qquad \frac{\vdash \varphi_1, \ldots, \varphi_j}{\vdash \bigcirc\varphi_1, \ldots, \bigcirc\varphi_j, p_1, \ldots, p_k}$$

- Step 2: connect each ψ to the formula where it comes from

Question: Is φ Valid?

Construction of Gentzen-style Proof:

- start at bottom with $\vdash \varphi$
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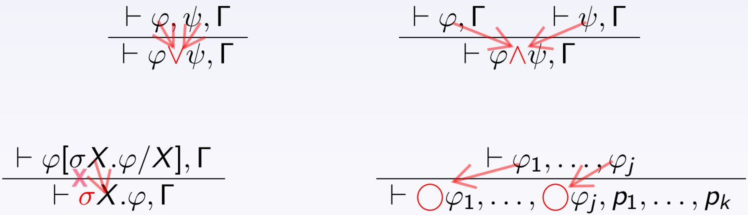

$$\frac{\vdash \varphi, \psi, \Gamma}{\vdash \varphi \lor \psi, \Gamma} \qquad \frac{\vdash \varphi, \Gamma \qquad \vdash \psi, \Gamma}{\vdash \varphi \land \psi, \Gamma}$$

$$\frac{\vdash \varphi[\sigma X.\varphi/X], \Gamma}{\vdash \sigma X.\varphi, \Gamma} \qquad \frac{\vdash \varphi_1, \ldots, \varphi_j}{\vdash \bigcirc\varphi_1, \ldots, \bigcirc\varphi_j, p_1, \ldots, p_k}$$

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Proof Tree Construction

Example

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\vdash \ldots}{\vdash \nu X.\sim}}{\vdash \bigcirc \nu X.\sim}}{\vdash p\,,\quad \bigcirc\bigcirc \nu X.\sim}}{\vdash (p \vee \bigcirc\bigcirc \nu X.\sim)} \qquad \dfrac{\dfrac{\dfrac{\vdash \ldots}{\vdash \mu Y.\sim}}{\vdash \neg p\,,\quad \bigcirc \mu Y.\sim}}{\vdash (\neg p \vee \bigcirc \mu Y.\sim)}}{\vdash (p \vee \bigcirc\bigcirc \nu X.\sim) \wedge (\neg p \vee \bigcirc \mu Y.\sim)}}{\vdash \nu X.(p \vee \bigcirc\bigcirc X) \wedge (\neg p \vee \bigcirc \mu Y.\sim)}}{\vdash \mu Y.\nu X.(p \vee \bigcirc\bigcirc X) \wedge (\neg p \vee \bigcirc Y)}$$

Proof Tree Construction

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Proof Tree Construction

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Proof Tree Construction

Example

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Proof Tree Construction

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Proof Tree Construction

Example

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Proof Tree Construction

Example

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Example

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  \qquad\qquad
  \dfrac{
    \dfrac{
      \dfrac{\vdash \ldots}{\vdash \mu Y.\sim}
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  \dfrac{
    \vdash (p \vee \bigcirc \bigcirc \nu X.\sim) \wedge (\neg p \vee \bigcirc \mu Y.\sim)
  }{
    \dfrac{\vdash \nu X.(p \vee \bigcirc \bigcirc X) \wedge (\neg p \vee \bigcirc \mu Y.\sim)}{\vdash \mu Y.\nu X.(p \vee \bigcirc \bigcirc X) \wedge (\neg p \vee \bigcirc Y)}
  }
}
$$

Proof Tree Construction

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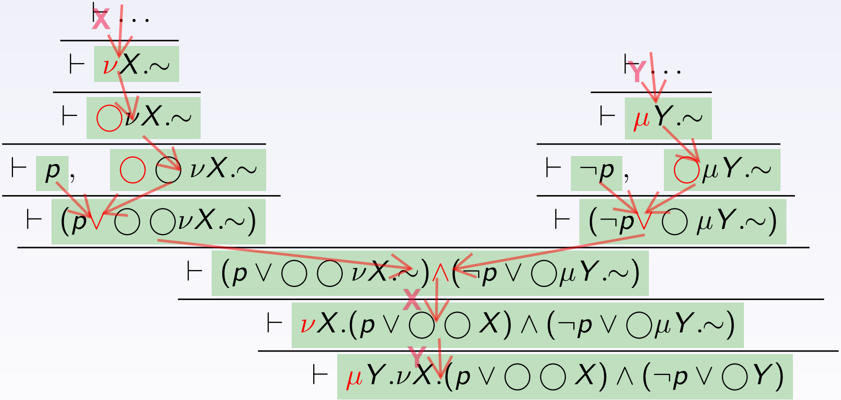
Finding Threads

Example



$\vdash \ldots$

$\vdash \nu X.\sim$

$\vdash \bigcirc \nu X.\sim$

$\vdash p, \quad \bigcirc \bigcirc \nu X.\sim$

$\vdash (p \vee \bigcirc \bigcirc \nu X.\sim)$

$\vdash \ldots$

$\vdash \mu Y.\sim$

$\vdash \neg p, \quad \bigcirc \mu Y.\sim$

$\vdash (\neg p \vee \bigcirc \mu Y.\sim)$

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$\vdash \mu Y.\nu X.(p \vee \bigcirc \bigcirc X) \wedge (\neg p \vee \bigcirc Y)$

Threads and Validity

Threads:

- thread = sequence of connected formulas, e.g. red lines on previous slide
- ν-thread = thread + outermost fixpoint that occurs ∞-often is of type ν, e.g. the left line with variable X on previous slide

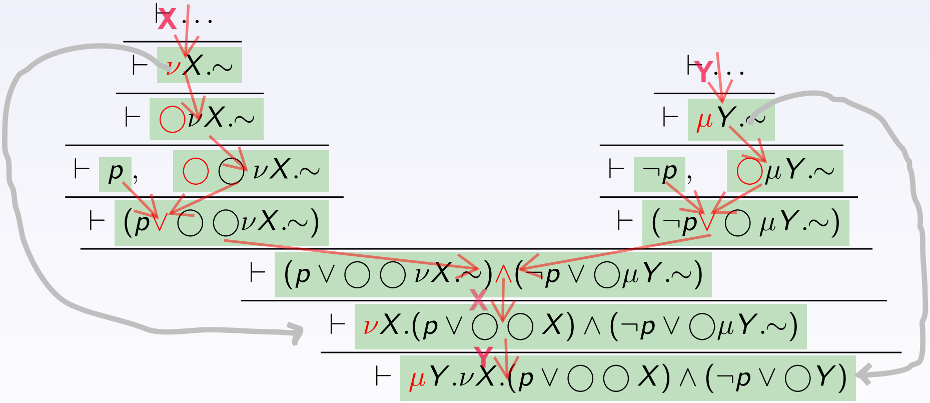
Theorem

root formula φ valid $\quad\Leftrightarrow$

- *each finite branch ends with $\vdash p, \neg p, \Gamma$, and*
- *each ∞-branch has ν-thread*

ν-Thread Example

Example



$\vdash \dots$

$\vdash \nu X.\sim$

$\vdash \bigcirc \nu X.\sim$

$\vdash p, \quad \bigcirc\bigcirc \nu X.\sim$

$\vdash (p \vee \bigcirc\bigcirc \nu X.\sim)$

$\vdash \dots$

$\vdash \mu Y.\sim$

$\vdash \bigcirc \mu Y.\sim$

$\vdash \neg p, \quad \bigcirc \mu Y.\sim$

$\vdash (\neg p \vee \bigcirc \mu Y.\sim)$

$\vdash (p \vee \bigcirc\bigcirc \nu X.\sim) \wedge (\neg p \vee \bigcirc \mu Y.\sim)$

$\vdash \nu X.(p \vee \bigcirc\bigcirc X) \wedge (\neg p \vee \bigcirc \mu Y.\sim)$

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Two Different Approaches

We developed two algorithms:

1. first algorithm is automata-based
2. second algorithm is relation-based

Automata Based Approach

Conceptually, we follow Vardi's approach, but

- our underlying proof-tree representation is more simple
- we give explicit constructions for automata

Construction:

1. Büchi automaton $\mathcal{A}$ that accepts branches of proof tree of φ
2. Büchi automaton $\mathcal{A}_\nu$ that accepts ν-thread branches

Lemma

$$\varphi \textit{ valid} \quad \Leftrightarrow \quad L(\mathcal{A}) \subseteq L(\mathcal{A}_\nu) \quad \Leftrightarrow \quad L(\mathcal{A}) \cap \overline{L(\mathcal{A}_\nu)} \neq \emptyset.$$

Complementation costly: Emptiness check in $2^{O(|\varphi|^2 log\, |\varphi|)}$.

Relation Based/Direct Approach

Step 1:

- Construct proof tree, represented as finite graph (infinite branches become loops).

Step 2:

- Let "$\dfrac{\Gamma}{\Delta}\; r$" rule application in proof. (nodes Γ, Δ = set of formulas)
- Save dependencies in relations: $R_{\Gamma,r,\Delta} \subseteq \Gamma \times \Delta \times \mathit{Vars}$

$(\varphi, \psi, X) \in R_{\Gamma,r,\Delta}$ $\Leftrightarrow$ red arrow $\dfrac{\vdash \ldots, \underset{\downarrow}{\varphi}, \ldots}{\vdash \ldots, \psi, \ldots}\; r$ in proof.

Relation Based/Direct Approach

...

- If we have $R_{\Gamma,r_1,\Delta}$ and $R_{\Delta,r_2,E}$ then we can calculate $R_{\Gamma,r_1r_2,E}$. (Connecting dependencies + preserving greater variables)
- Calculate transitive closure.

Step 3:

- For each node Γ: if $R_{\Gamma,\pi,\Gamma} \circ R_{\Gamma,\pi,\Gamma} = R_{\Gamma,\pi,\Gamma}$ we have loop (=infinite branch) along π^ω where $\pi = (r_1 r_2 \dots r_n)$.
- if $R_{\Gamma,\pi,\Gamma}$ contains arrow $(\varphi, \varphi, X_\nu)$ then ν-thread branch!
- Check that all loops are ν-thread branches.

Application of SCT Algorithm

Size Change Termination (SCT)

- Algorithm checks whether a functional program terminates
- Proposed by Lee, Jones, Ben-Amran
- Our algorithm is an instance of SCT

Analogies:

- function symbols $f, g, h, \ldots$ $\approx$ nodes
- function calls $\approx$ rule application on premises
- combination of function calls $fgghf \ldots$ $\approx$ branch in tree
- decreasing measure for function parameters $\approx$ unfolding of greatest fixpoint

Complexity for computing transitive closure: $2^{O(|\varphi|^3)}$

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Benchmark Formulas

Two families of formulas:

- *Include$_n$*: $((pp)^n q)^\omega \subseteq ((pp)^* q)^\omega$ (valid)

$$\nu X.(\underbrace{p \wedge O(p \wedge O(\ldots O(p}_{\text{2n times}} \wedge O(\neg p \wedge OX))\ldots)))$$

$$\rightarrow \quad \nu Z.\mu Y.(p \wedge O(p \wedge OY) \vee (\neg p \wedge OZ))$$

- *Counter$_n$*: n-bit counter (not valid)
  smallest countermodel needs $2^{|\varphi|}$ states

Comparison of the Two Algorithms

	Include$_n$		*Counter$_n$*	
n	Auto.	Rel.	Auto.	Rel.
0	0	0	0	0
1	0	2	0	0
2	1	5	3	2
3	1	10	36	50
4	3	18	489	1131
5	4	31	5694	†

- numbers denote run-times in seconds
- on our formulas both algorithms performs the same

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Summary

- Simple Gentzen-style proof system for validity
- Automata-based decision procedure in $2^{O(|\varphi|^2 log\, |\varphi|)}$
- Application of SCT to effective proof search in $2^{O(|\varphi|^3)}$
- First implementation of μTL validity-checker

Ongoing/future work

- Application to modal μTL
- Improvements to algorithms
- Evaluation of implementation (e.g. comparing with LTL model-checkers)